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AUTHOR	Dziuban, Charles D.; Shirkey, Edwin C.
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ABSTRACT

Three techniques for assessing the adequacy of correlation matrices for factor analysis were applied to four examples from the literature. The methods compared were: (1) inspection of the off diagonal elements of the anti-image covariance matrix S(to the 2nd) R(to the -1) and S(to the 2nd); (2) the Measure of Sampling Adequacy (M.S.A.), and (3) Bartletts Test of Sphericity. Of the four matrices used for the study, two were comprised of eight variables and one each of fourteen and twenty. The sample sizes ranged from 50 to over 3,000. The results indicated that the three methods for overall assessment yeidlded comparable results. It was recommended, however, that methods for individual variables assessment also be used. (Author)

ON THE ASSESSMENT OF PSYCHOMETRIC ADEQUACY IN CORRELATION MATRICES

by

Charles D. Dziuban

and

Edwin C. Shirkey

Florida Technological University

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Background and Methods:

Recently attention has been focused upon the psychometric assessment of correlation matrices prior to the application of factor analysis (Tobias and Carlson 1969, Cooley and Lohnes 1970, Kaiser 1971, Dziuban and Harris 1972). The practice is often recommended (Tobias and Carlson 1969, Cooley and Lohnes 1971) but the results of such procedures are rather infrequently found in the literature. It was the purpose of this study to empirically compare the results of such techniques when applied to four well known correlation matrices.

The initial procedure involved the application of Bartlett's Test of Sphericity. Explanations of the test may be found in Kendall (1957), Anderson (1958), Cooley and Lohnes (1971). Maxwell (1959), and Tobias and Carlson (1969) have recommended the test be used by psychologists prior to the application of factor analysis. It is computed by the formula:

$$-\left[(N-1)-1/6(2P+5\right] \log_e R$$

where N is the sample size, P is the number of variables and R is the determinant of the correlation matrix. For large N the statistic is approximately distributed as Chi Square with 1/2 P (P-1) degrees of freedom and has the associated hypothesis that the sample correlation matrix came from a multivariate normal population in which the variables of interest are independent.

Knapp and Swoyer (1967) conducted a study of the power of the Bartlett test and found it to be quite substantial. They determined that for N=200, P=10 and α=.05 one would be virtually certain to reject the hypothesis stated above when the correlations were as low as .09. This seems to suggest that the power of the test is highly sensitive to the sample size.

The second procedure involved the inspection of the off-diagonal elements of the anti-image covariance matrix $S^2R^{-1}S^2$: where R^{-1} is the inverse of the correlation matrix. The matrix S^2 is defined as $\left[\text{diag } R^{-1}\right]^{-1}$ (Guttman 1953). Kaiser (1963) has summarized this practice as follows:

"The preceding material suggests that G, the image covariance matrix, might well be a good approximation to $R-U^2$, the so called reduced correlation matrix" (actually the covariance matrix of the common parts of the tests). How can we tell if this approximation is good? Most simply by looking at the off-diagonal elements of the anti-image covariance matrix Q (or $S^2R^{-1}S^2$) ... In this case if our N is essentially infinate, we have a comprehensive selection of tests from the universe of tests. If on the other hand, Q is not near-diagonal, we know that the approximation is poor. However, when this occurs, we have evidence that factor analysis is not appropriate for the data at hand. We may not have thoroughly covered the universe under consideration or that the factor analytic model may not even apply as $N \to \infty$."

The third procedure involved the computation of the Measure of Sampling Adequacy - M. S. A. (Kaiser 1970) for each matrix. The index is defined as:

$$\text{M.S.A.} = 1 - \frac{\sum_{j \neq} \sum_{k} g^2_{jk}}{\sum_{j \neq} \sum_{k} r^2_{jk}}$$

where the g^2's are the squares of the off-diagonal elements of the anti-image correlation matrix $SR^{-1}S$ and the r^2's are the squares of the original correlations.

The index yields an assessment of whether the variables belong together psychometrically and thus whether the matrix is appropriate for factor analysis. A similar measure may be defined for each variable separately:

$$\text{M.S.A.}(J) = 1 - \frac{\sum_{\substack{k \\ j \neq k}} q_{jk}^2}{\sum_{\substack{k \\ j \neq k}} r_{jk}^2}$$

It gives an indication of whether a particular variable j "belongs to the family" psychometrically. Kaiser (1970) indicated that any M.S.A. lies between minus infinity and plus one. He further indicated that the index appears to be a function of four "main effects" and that holding the others constant it improves as:

1. The number of variables increases.
2. The (effective) number of factors decreases.
3. The number of subjects increases.
4. The general level of correlation increases.

Those three procedures were applied to four well known correlation matrices from the literature:

1. The eight physical variables N=305 , (Harman 1967)
2. The eight political variables N=147 , (Harman 1967)
3. The Shaycroft Matrix N=3689 , (Dziuban and Harris, 1972)

   This matrix was based upon ten "Project Talent" variables of interest and four random deviates.
4. The Armstrong and Soelberg Matrix N=50 (Armstrong and Soelberg, 1968)

   This matrix was entirely based upon correlations among 20 variables which were random normal deviates.

Results:

The Bartlett Test lead to a clear rejection of the hypothesis of interest for three of the four matrices -- the physical variables $|R| = .98 \times 10^{-3}$, the political variables $|R| = .17 \times 10^{-3}$ and the Shaycroft Matrix $|R| = .83 \times 10^{-3}$. The Armstrong Soelberg Matrix, however, exhibited an exact probability of .55, $|R| = .011$.

The anti-image covariance matrices $S^2R^{-1}S^2$ are presented in tables one through four. Elements which were not zero to the first decimal place were considered to be contributing to the non-diagonality of the matrices. The physical variables produced eight off-diagonal elements of $S^2R^{-1}S^2$ which were non zero (Table I)$\approx$14%. The political variables produced four such elements (7%) (Table II) while the Shaycroft Matrix yielded ten non zero elements (Table III)$\approx$5.5%. The Armstrong and Soelberg Matrix exhibited an anti-image covariance matrix with 128 non zero off-diagonal elements (Table IV)$\approx$33%.

The results of the application of the measure of Sampling Adequacy (M.S.A.) of the correlation matrices are presented in Table V. According to Kaiser's present calibration:

In the .90's, Excellent
In the .80's, Good
In the .70's, Fair
In the .60's, Poor
Below .60 , Terrible.

The physical and political variables yielded values which would make them at least appropriate for factor analysis, .83 and .77. The Shaycroft Matrix yielded an overall M.S.A. of .91 which puts it in the excellent range while the Armstrong and Soelberg Matrix yielded an M.S.A. value of -.38 which should be clear indication that the matrix should not be factor analyzed.

Summary:

We have applied three tests for psychometric adequacy to four well known correlation matrices ~~from the~~ literature. Those matrices represented somewhat disparate conditions under which factor analysis has been utilized. They ranged from sample size of 50 to over three thousand. The Armstrong and Soelberg Matrix was based upon the correlations of random numbers while the Shaycroft data were predominately composed of variables from "Project Talent." We have cited the results of the tests and have reached some speculative conclusions.

Initially, the power of the Bartlett Test seems quite sensitive to the sample size. Knapp and Swoyer have provided evidence which indicates that this is true. We further suggest, however, that rejection of the hypothesis that the sample came from a population in which the variables of interest are independent may not be sufficient evidence that the data at hand are appropriate for factor analysis. It seems possible that erroneous results might still be obtained by routinely using the Bartlett Test and then principal components - for instance, in the case of the Shaycroft Matrix. Four of the fourteen variables were random, yet were not detected or evidenced by the overall rejection of the hypothesis.

The anti-image covariance matrix ($S^2R^{-1}S^2$) allows comparable decisions. The most nearly diagonal matrix was the Shaycroft which was heavily dominated by the "TALENT" variables. The Armstrong and Soelberg Matrix yielded approximately one third of its elements as non-zero. Some appropriate decision rule seems necessary, however -- that is how diagonal is "diagonal?" It seems to us that one gets a feel for his data by examining the off-diagonal elements of $S^2R^{-1}S^2$ but that it becomes difficult to make a decision except in extreme cases.

M. S. A. offers advantages which the previous two procedures lack. Kaiser has least provided some "gut level" decisions rules for the overall quality of the data. The second advantage of M.S.A. lies in the fact that it provides specific assessment of individual variables. For instance, the random variables were readily identified in the Shaycroft Matrix. Since it is usually assumed that one makes some apriori judgements about the variables which should be incorporated into factor analytic investigation M.S.A. might be the logical intermediate step to assess the quality of those apriori judgements. Prior use of that procedure might increase the opportunity of realizing "interpretable" factors.

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TABLE I

ANTI IMAGE COVARIANCE MATRIX ($S^2R^{-1}S^2$) PHYSICAL VARIABLES

Variable							
1	--						
2	-.05						
3	-.02	-.10					
4	-.10	-.03	-.04				
5	-.04	.04	-.02	-.01			
6	-.03	.00	-.01	.04	-.16		
7	.04	-.02	.03	.04	-.17	-.02	
8.	.02	-.07	.02	.01	108	.08	-.08

TABLE II

ANTI IMAGE COVARIANCE MATRIX ($S^2R^{-1}S^2$) POLITICAL VARIABLES

Variables							
1	--						
2	<u>-.11</u>						
3	.06	-.07					
4	-.03	.01	-.00				
5	-.00	.03	.06	-.06			
6	.04	-.05	-.02	.03	-.03		
7	.02	-.02	.05	-.05	<u>.19</u>	.01	
8	.04	-.00	.04	-.09	.05	.04	-.00

TABLE III

ANTI IMAGE COVARIANCE MATRIX ($S^2R^{-1}S^2$) - SHAYCROFT MATRIX

Variables													
1	--												
2	-.09												
3	-.03	-.03											
4	-.06	-.06	-.07										
5	-.13	.03	.01	-.08									
6	-.04	-.04	.01	.06	-.00								
7	-.07	-.10	.00	-.06	-.01	-.09							
8	.00	.04	-.02	-.02	-.12	-.03							
9	-.00	.01	-.01	-.02	.01	-.04	-.08	-.21					
10	.01	.01	-.14	-.03	.02	-.09	-.01	-.02	-.03				
11	-.00	.00	.00	-.01	-.00	.00	.01	.01	.00	.00			
13	.00	.00	.00	.00	-.01	.01	.01	-.01	-.02	-.01	.01	-.02	
14	.01	-.01	-.01	.00	-.00	.00	-.05	-.00	.00	.01	.03	.04	-.01

TABLE IV

ANTI IMAGE COVARIANCE MATRIX ($S^2R^{-1}S^2$) - ARMSTRONG AND SOELBERG MATRIX

Variable																			
1	--																		
2	-.12																		
3	.05	-.04																	
4	.19	-.19	.21																
5	.10	.04	.05	-.02															
6	.03	-.07	.18	.17	.03														
7	-.10	.06	-.22	-.13	.04	-.01													
8	.04	.00	-.02	.03	-.02	-.12	-.06												
9	-.02	-.20	-.07	.05	.01	-.03	.13	-.02											
10	.04	.01	.03	.02	.15	.13	.07	.06	-.06										
11	-.00	.04	-.12	-.14	-.07	-.12	.15	-.06	.04	-.09									
12	-.04	-.01	-.19	-.16	.01	-.11	.26	-.17	.15	-.05	.14								
13	.04	.11	.02	.01	-.08	-.11	-.09	.13	-.16	-.08	.11	-.08							
14	.13	.10	-.07	-.13	-.05	-.11	-.02	.02	-.11	-.23	.15	.00	.18						
15	.06	-.07	.03	.06	-.01	.07	.01	.01	-.06	.08	.01	-.02	-.00	.09	-.07				
16	.03	.13	.05	-.06	.08	.02	.03	-.21	.04	-.02	-.06	.11	-.19	-.02	-.11				
17	-.03	-.04	-.03	-.02	-.17	-.14	.00	-.00	.06	-.20	.05	.05	-.00	.02	.13	.05			
18	.17	-.16	-.05	.06	-.00	-.00	.06	-.11	.18	-.10	-.01	.11	-.08	-.14	.16	.10	.22		
19	-.11	.02	-.20	-.13	-.00	-.04	.06	-.13	.05	-.13	.00	.14	.17	.05	.02	.03	.09		
20	-.01	-.12	.04	.07	-.07	.20	-.08	.02	-.11	-.00	-.07	-.05	-.03	.02	-.13	-.10	-.17	-.08	.05

TABLE V

MEASURES OF SAMPLING ADEQUACY (OVERALL AND INDIVIDUAL)

PHYSICAL VARIABLES (OVERALL M.S.A. = .83)

VARIABLE	INDIVIDUAL M.S.A.
1	.86
2	.78
3	.85
4	.87
5	.75
6	.82
7	.80
8	.87

POLITICAL VARIABLES (OVERALL M.S.A. = .77)

VARIABLE	INDIVIDUAL M.S.A.
1	.62
2	.68
3	.81
4	.85
5	.12
6	.92
7	.72
8	.83

SHAYCROFT MATRIX (OVERALL M.S.A. = .91)

	VARIABLE	INDIVIDUAL M.S.A.
	1	.92
	2	.93
	3	.87
	4	.94
"Talent"	5	.89
Variables	6	.93
	7	.93
	8	.87
	9	.92
	10	.86
	11	.68
Random	12	.47
Variables	13	.43
	14	-.83

ARMSTRONG AND SOELBERG MATRIX (OVERALL M.S.A. = -.38)

VARIABLE	INDIVIDUAL M.S.A.
1	-.03
2	.04
3	-1.41
4	-2.11
5	-.30
6	-.34
7	-.22
8	-.28
9	-.78
10	-.38
11	-1.01
12	-.75
13	-.29
14	-.27
15	-.14
16	.20
17	-.45
18	-.49
19	-.33
20	-.11